Problem 12) This is the same integral as in Problem 11, namely, $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$, with $a = 1 + t^2$ and $b = -2t$. The condition $a > |b|$ is automatically satisfied because

$$a > |b| \quad \rightarrow \quad 1 + t^2 > 2|t| \quad \rightarrow \quad 1 + t^2 - 2|t| > 0 \quad \rightarrow \quad (1 - |t|)^2 > 0.$$

The last inequality is always valid and, therefore, $a > |b|$. Using the result of Problem 11, we have

$$\int_0^{2\pi} \frac{d\theta}{1 - 2t\cos\theta + t^2} = \frac{2\pi}{\sqrt{a^2 - b^2}} = \frac{2\pi}{\sqrt{1+t^4+2t^2-4t^2}} = \frac{2\pi}{\sqrt{(1-t^2)^2}} = \frac{2\pi}{1-t^2}.$$

For $|t| > 1$, everything remains the same, except that, in the last step, $\sqrt{(1-t^2)^2} = t^2 - 1$. The integral will then be equal to $2\pi/(t^2 - 1)$.

If $|t| = 1$, we will have $a = |b|$. With reference to Problem 11, the roots of the denominator (z_1 and z_2) become equal to each other; both roots will be *on* the unit circle, and the method of integration used for this type of problem no longer works. Using a limit argument, we see that as $|t| \rightarrow 1$, the integral diverges to ∞.